MATH 2250 Calculus I
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Practice Quiz No. 3

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 State the following rules for derivatives:

a *The constant rule*

If $f(x) = c$, $f'(x) = 0$.

b *The constant multiple rule*

If $f(x) = c \cdot g(x)$, $f'(x) = c \cdot g'(x)$

c *The sum rule*

If $m(x) = f(x) + g(x)$, $m'(x) = f'(x) + g'(x)$

d *The product rule*

If $m(x) = f(x) \cdot g(x)$, $m'(x) = f'(x) g(x) + f(x) g'(x)$

e *The power rule*

If $f(x) = x^n$, $f'(x) = n x^{n-1}$

f *The quotient rule*

If $m(x) = \dfrac{f(x)}{g(x)}$, $m'(x) = \dfrac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$

Problem 2 Prove the constant rule.

We want to prove that: if $f(x) = c$, then $f'(x) = 0$.

Assuming $f(x) = c$, by the definition of $f'(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0$$

$= 0$, so $f'(x) = 0$. ∎

Problem 3 Prove the constant multiple rule.

We want to prove that: if $f(x) = c \cdot g(x)$, then $f'(x) = c \cdot g'(x)$. Assuming that $f(x) = c \cdot g(x)$, by the definition of $f'(x)$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{cg(x+h) - cg(x)}{h}$

$$= \lim_{h \to 0} c \left(\frac{g(x+h) - g(x)}{h} \right) = c \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = c\, g'(x)$$ ∎

Problem 4 Prove the sum rule.

We want to prove that: if $m(x) = f(x) + g(x)$, then $m'(x) = f'(x) + g'(x)$. Assuming that $m(x) = f(x) + g(x)$, by the definition of $m'(x)$, $m'(x) = \lim_{h \to 0} \frac{m(x+h) - m(x)}{h} =$

$$= \lim_{h \to 0} \frac{(f(x+h) + g(x+h)) - (f(x) + g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left(\left(\frac{f(x+h) - f(x)}{h} \right) + \left(\frac{g(x+h) - g(x)}{h} \right) \right)$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right) + \lim_{h \to 0} \left(\frac{g(x+h) - g(x)}{h} \right) = f'(x) + g'(x)$$ ∎

Problem 5 Find the derivative of the following function, remembering to show all of your work:

$$f(x) = 4x^3 - 5x^2 + 2x + 10$$

$$f'(x) = \frac{\partial}{\partial x}\left(4x^3 - 5x^2 + 2x + 10\right)$$
$$= \frac{\partial}{\partial x}(4x^3) + \frac{\partial}{\partial x}(-5x^2) + \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial x}(10)$$
$$= 4\frac{\partial}{\partial x}(x^3) + (-5)\frac{\partial}{\partial x}(x^2) + 2\frac{\partial}{\partial x}(x) + 0$$
$$= 4(3x^2) - 5(2x) + 2(1)$$
$$= 12x^2 - 10x + 2$$

Problem 6 Find the derivative of the following function, remembering to show all of your work:

$$f(x) = e^x(x^5 + \sqrt{x})$$

$$f'(x) = \frac{\partial}{\partial x}\left(f(x)\right) = \frac{\partial}{\partial x}\Big(\underbrace{e^x}_{h(x)}\underbrace{(x^5 + \sqrt{x})}_{g(x)}\Big)$$

so $h(x) = e^x$

$h'(x) = e^x$

$g(x) = x^5 + x^{1/2}$

$g'(x) = 5x^4 + \frac{1}{2}x^{-1/2}$

$$= h'(x)\,g(x) + h(x)\,g'(x)$$
$$= (e^x)(x^5 + x^{1/2}) + (e^x)(5x^4 + \tfrac{1}{2}x^{-1/2})$$

Don't bother simplifying beyond this point unless you have to use this result for something else!

Problem 7 Find the derivative of the following function, remembering to show all of your work:

$$f(x) = \frac{-x^{1/2}}{e^x + 2x}$$

$f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}\left(\frac{-x^{1/2}}{e^x+2x}\right)$, let $h(x) = -x^{1/2}$, $g(x) = e^x + 2x$,

so $h'(x) = -\frac{1}{2}x^{-1/2}$, $g'(x) = e^x + 2$

$= \frac{g(x)h'(x) - h(x)g'(x)}{(g(x))^2}$

$= \frac{(e^x+2x)\left(\frac{-1}{2}x^{-(1/2)}\right) - (-x^{-1/2})(e^x+2)}{(e^x+2x)^2}$

Don't bother simplifying beyond this point (unless you have to)

Problem 8 Find the derivative of the following function, remembering to show all of your work:

$$f(x) = e^x + x^e$$

$f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}(e^x + x^e)$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}(x^e)$

$= e^x + ex^{e-1}$

(Remember that e is just a constant)